Geometry and algorithms for orientations

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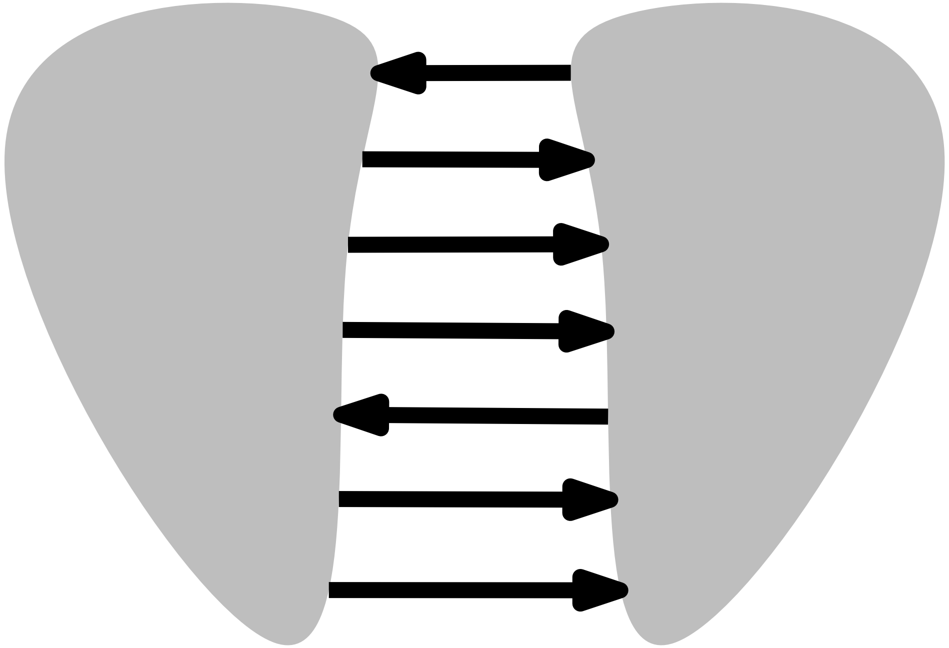
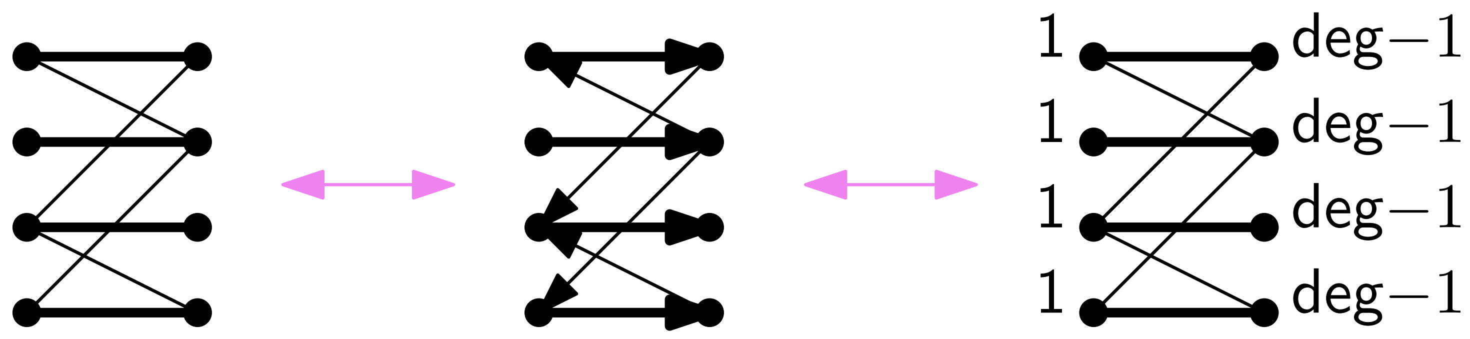


Orientations with prescribed outdegree

$G = (V, E)$, $\alpha \in \mathbb{N}^V \rightsquigarrow O_\alpha(G) = \{D \mid \underline{D} = G \text{ and } \delta^+(v) = \alpha(v) \forall_{v \in V}\}$

Example: perfect matchings of bipartite graphs



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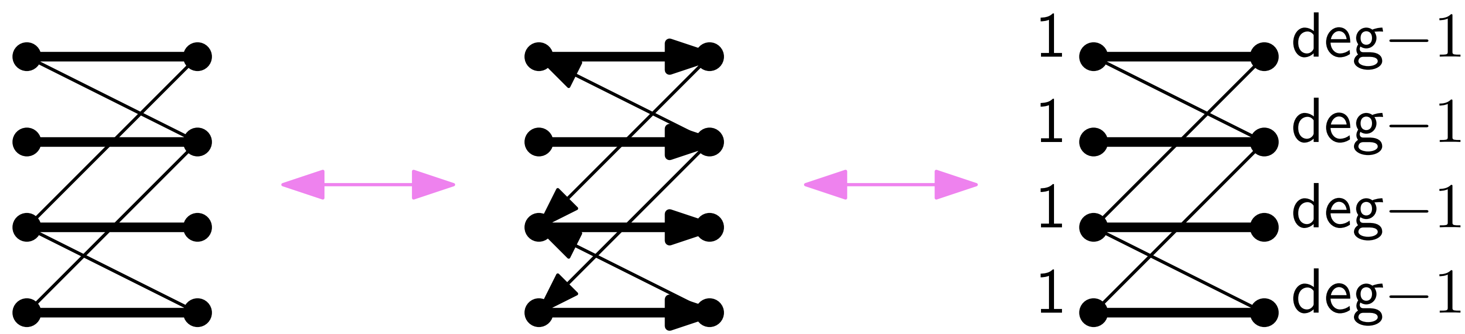
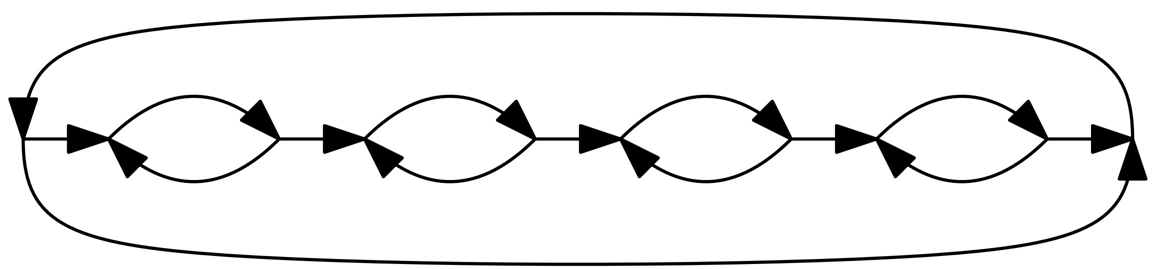
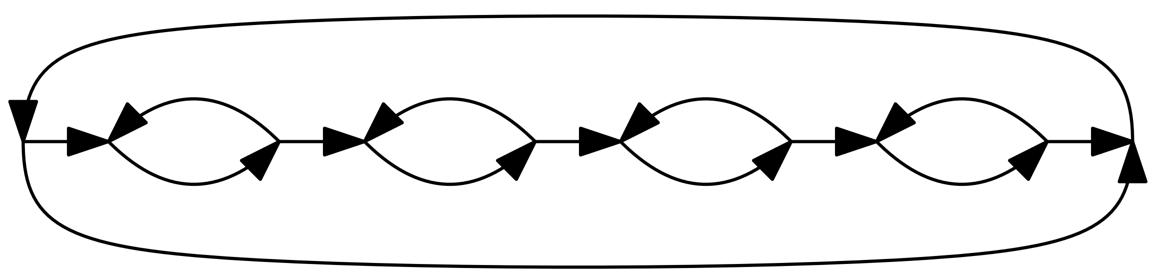


Obs: $O_\alpha(G)$ connected by flipping directed cycles ($D \setminus D'$ Eulerian)





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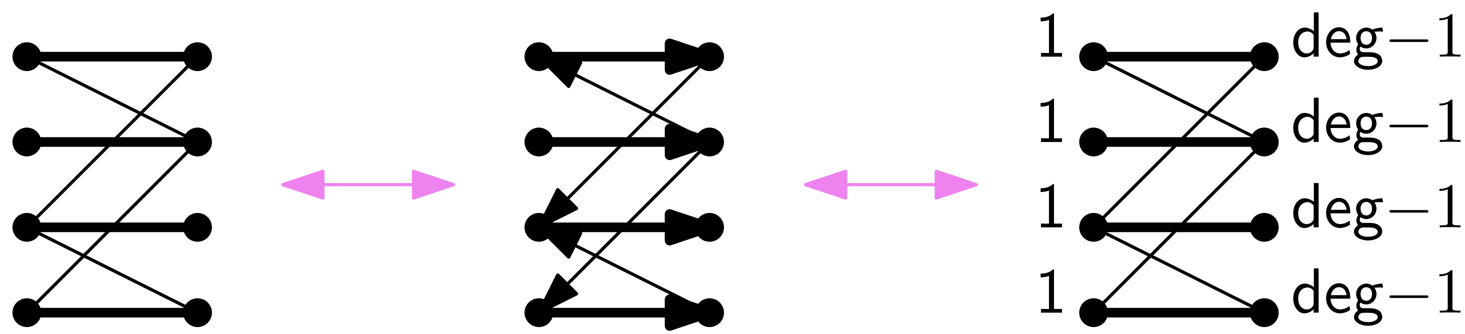

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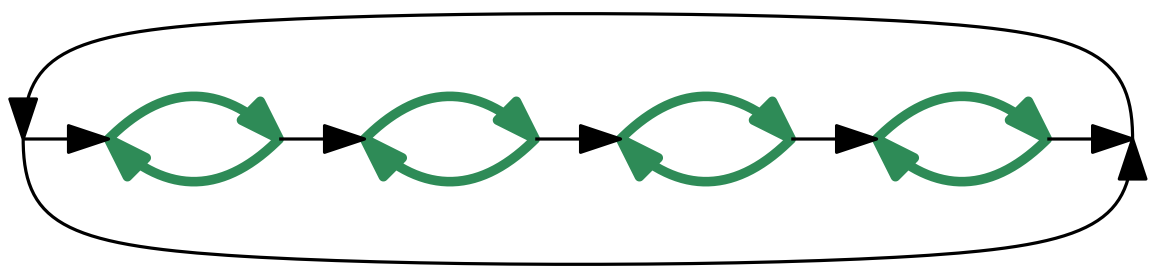

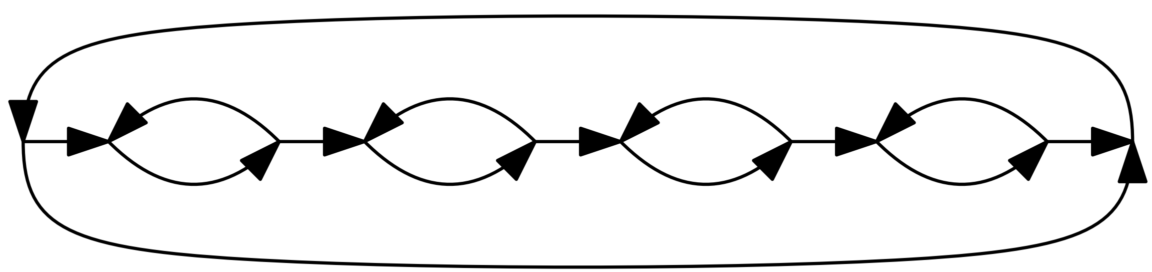

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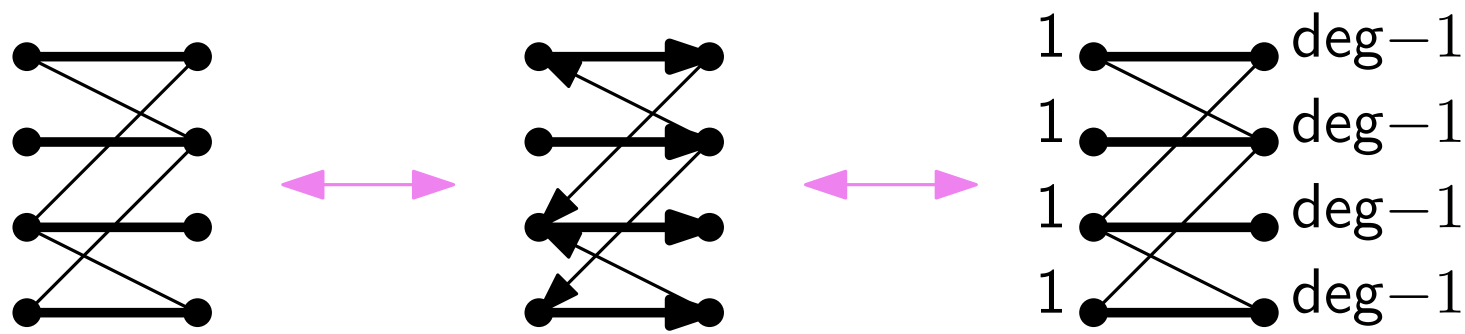

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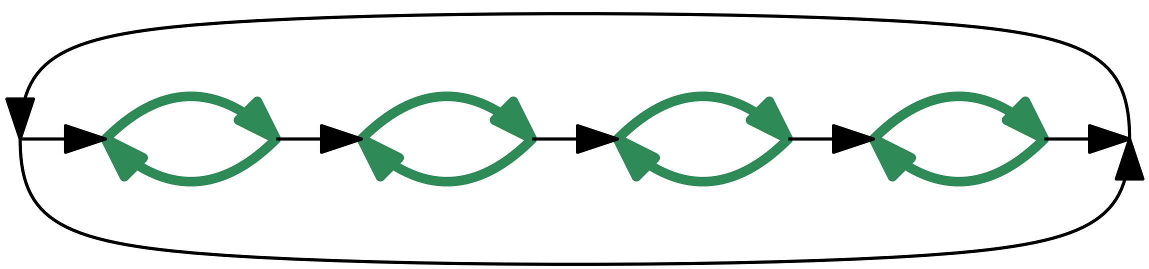

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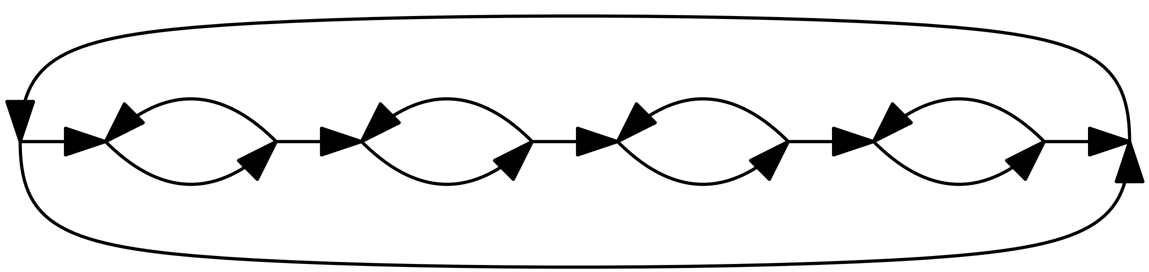

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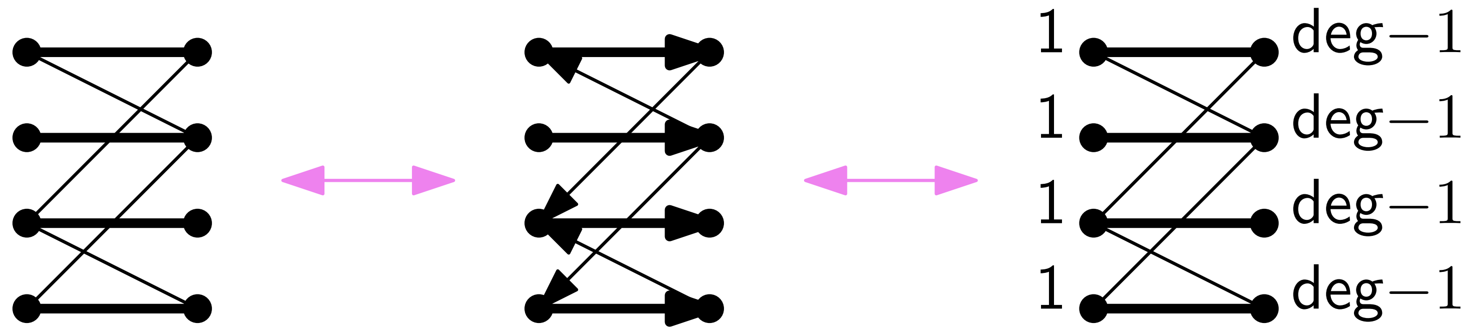

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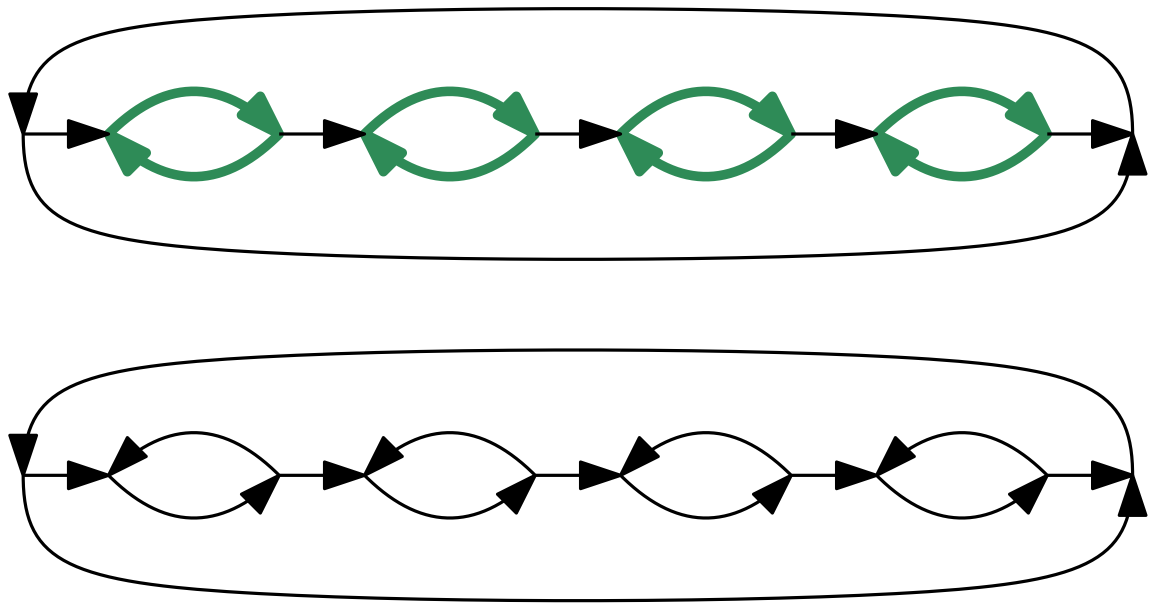

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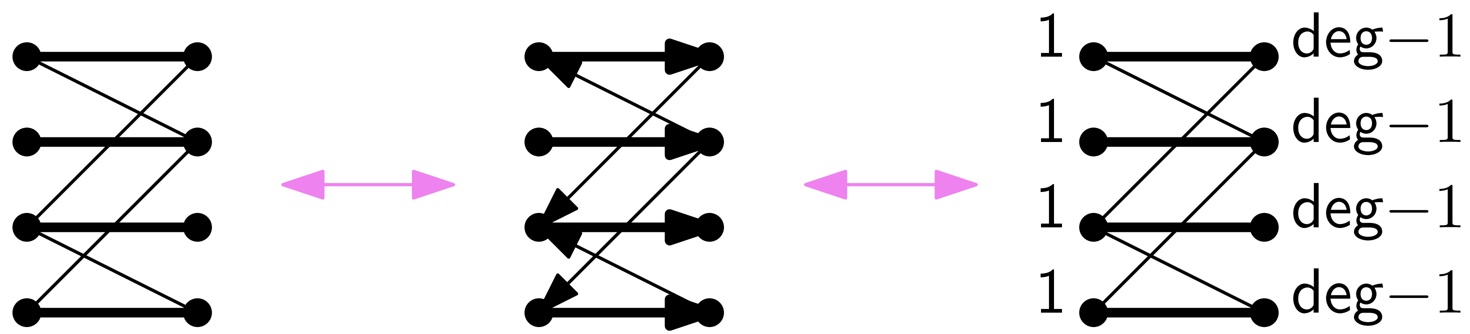

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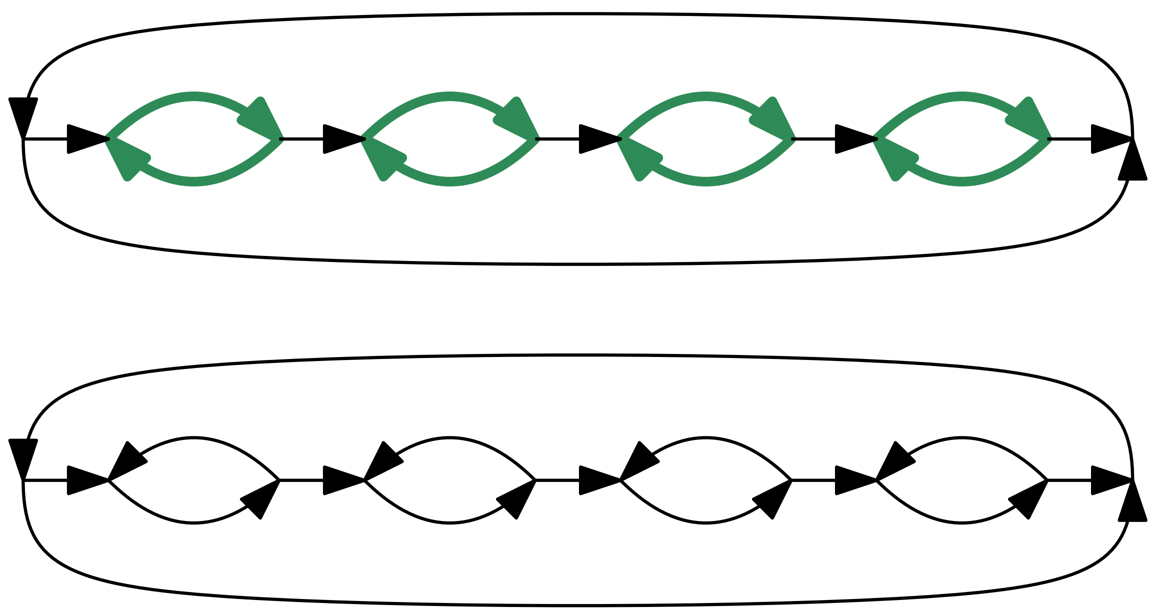

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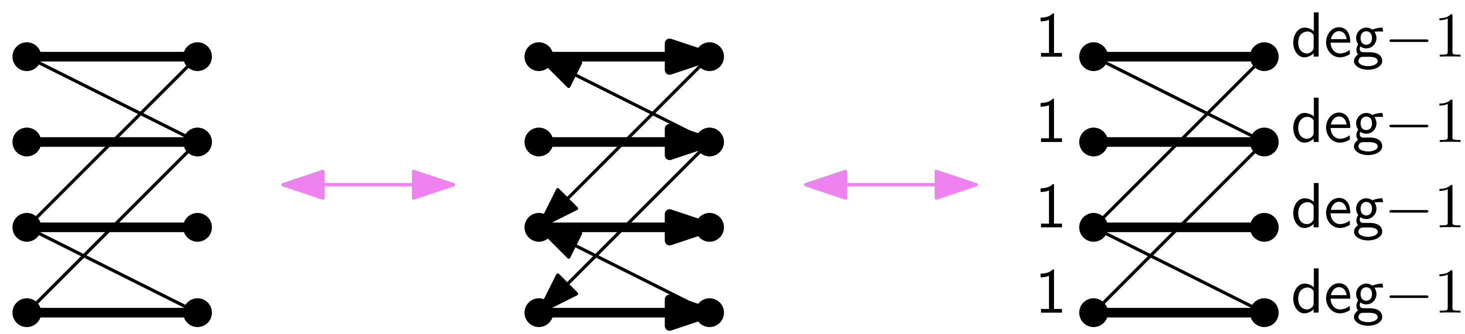

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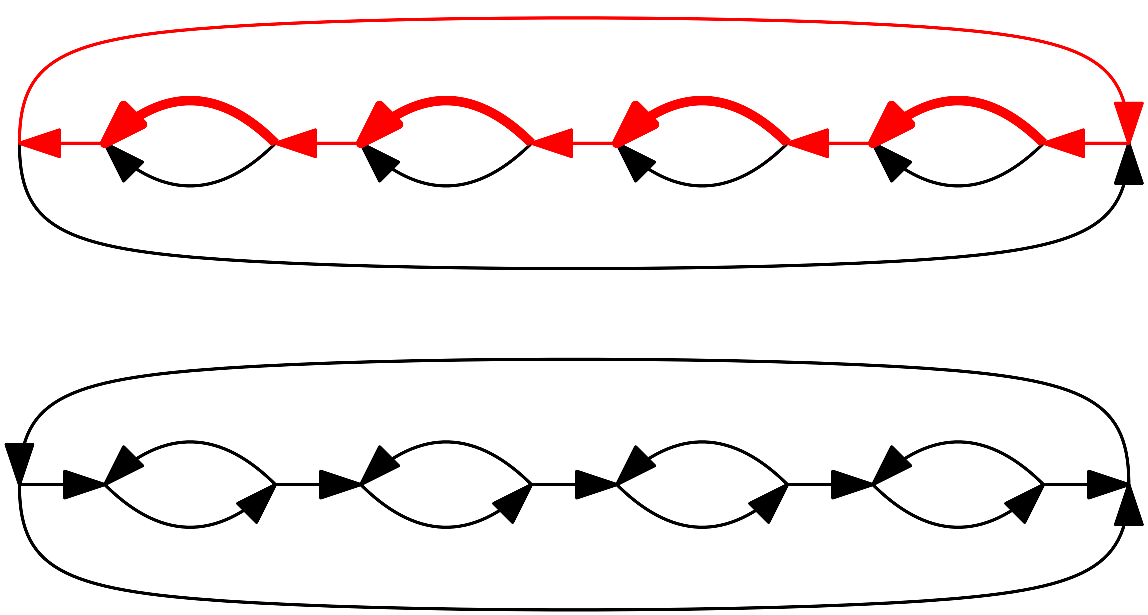

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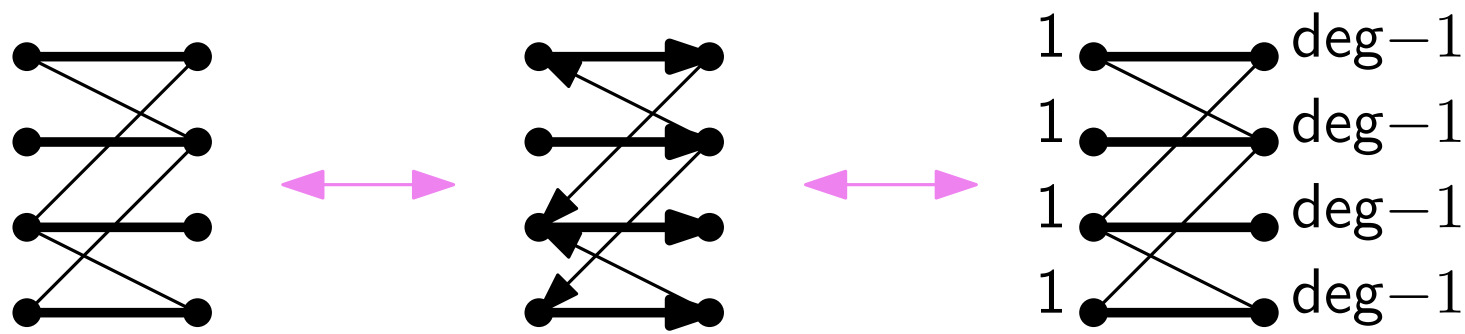

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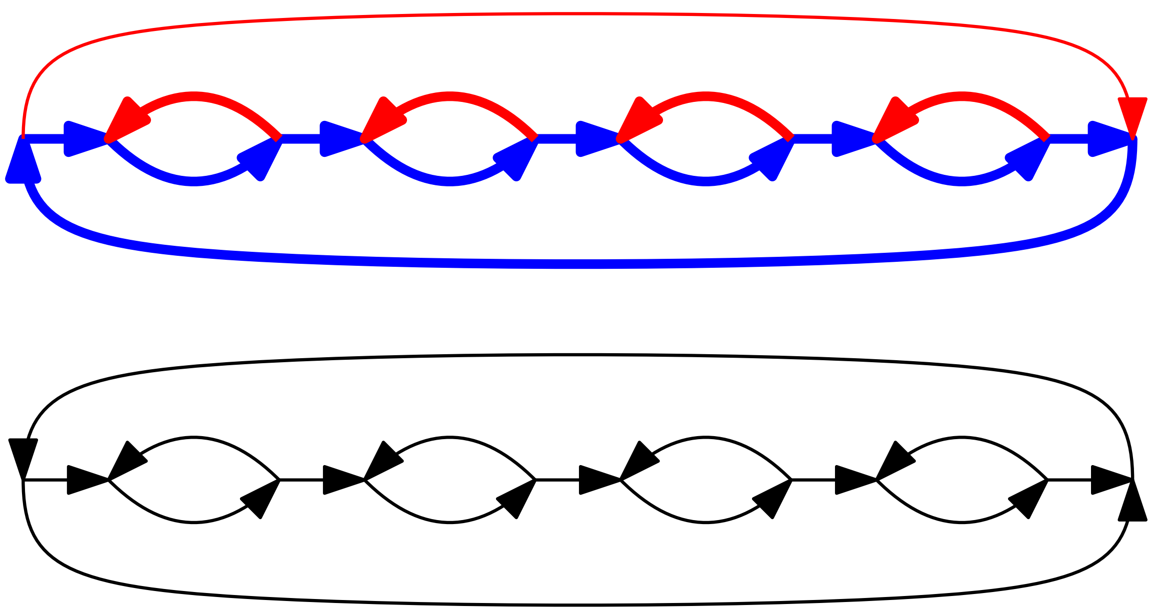

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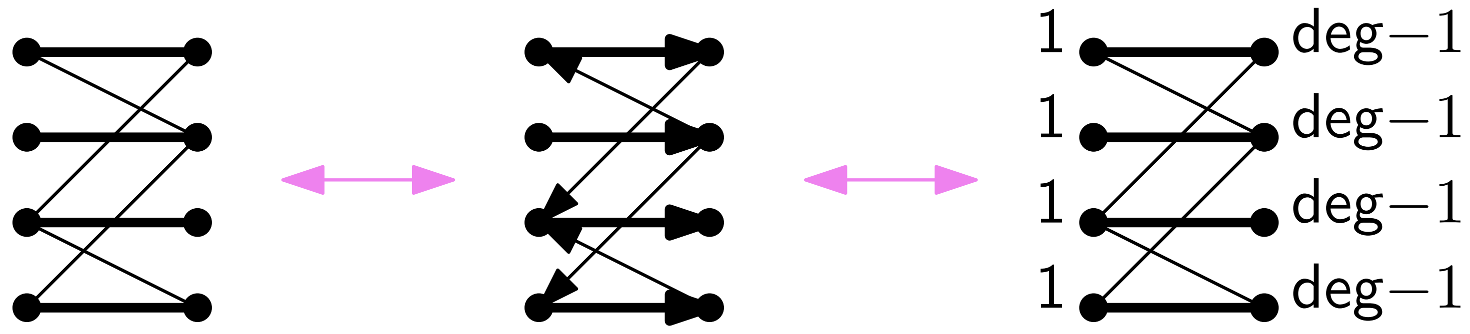

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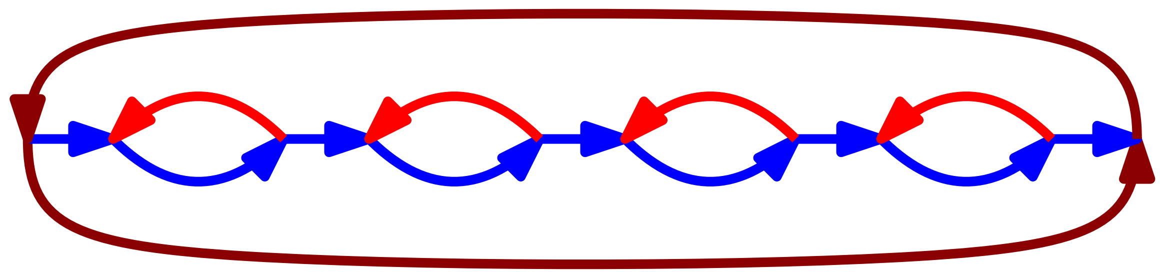

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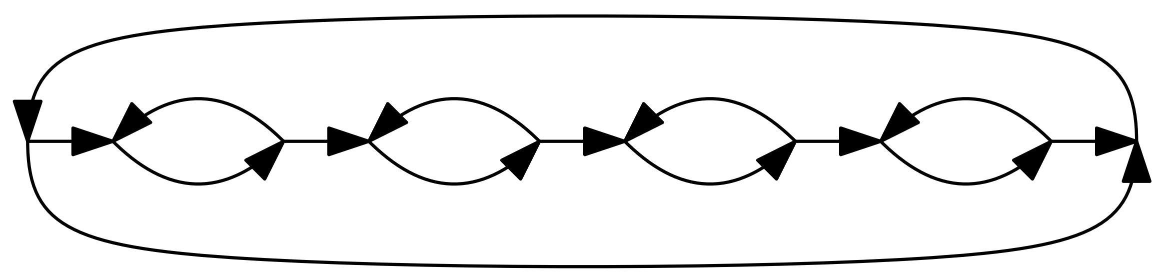
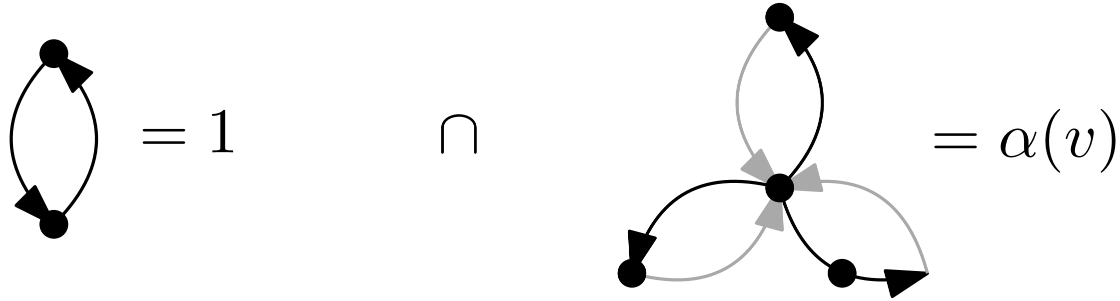
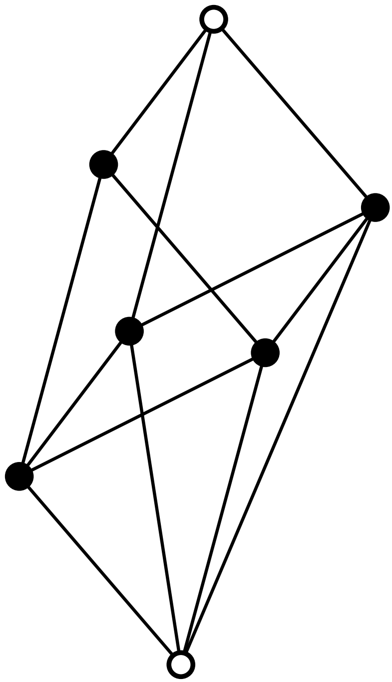


Distances in polytopes

$O_\alpha(G)$ is the base set of intersection of two partition matroids M_1, M_2.



Flip-graph on $O_\alpha(G)$ is the graph of the intersection P of the base polytopes of M_1, M_2.



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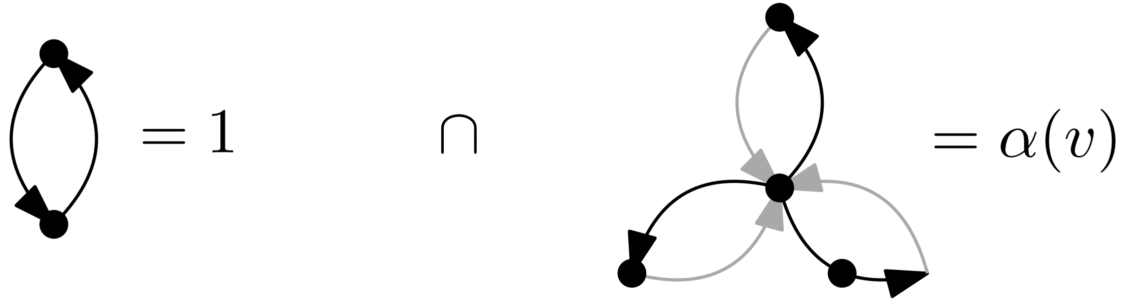

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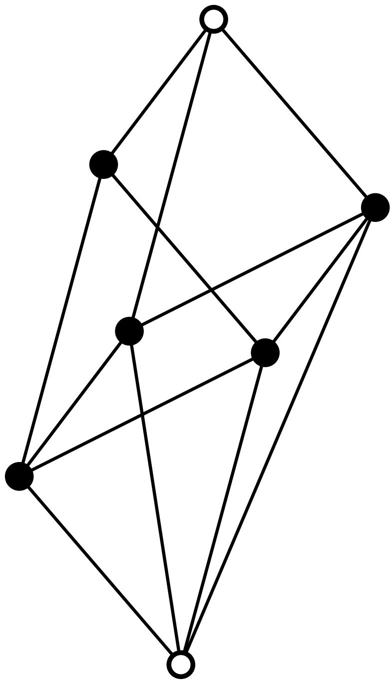

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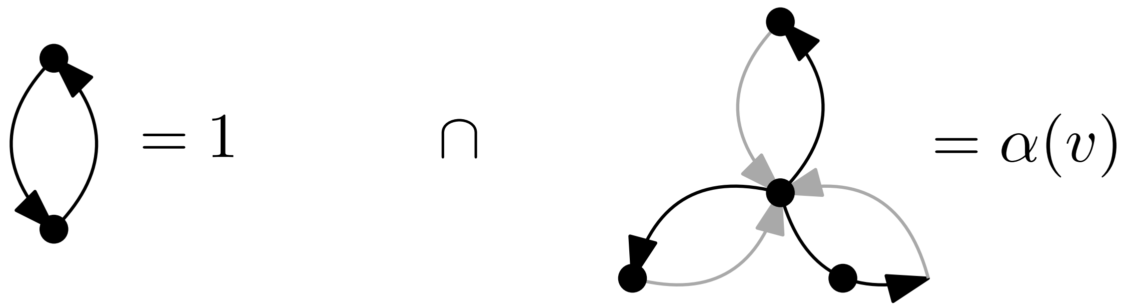

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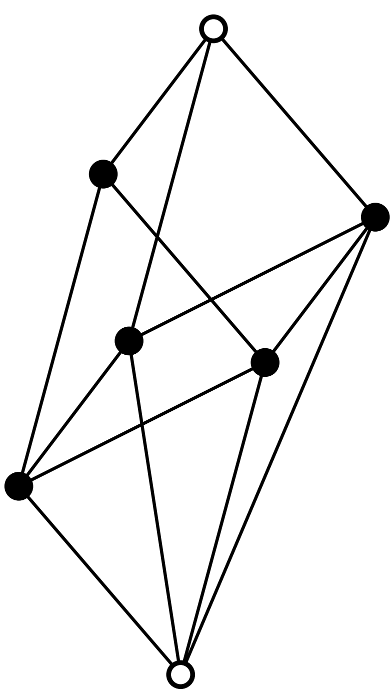

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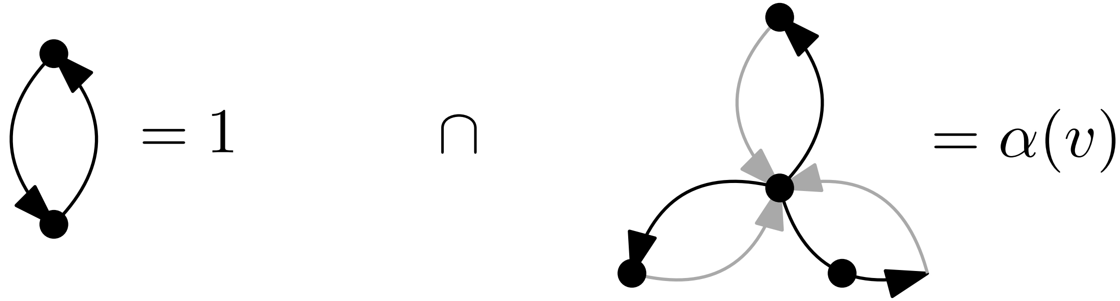

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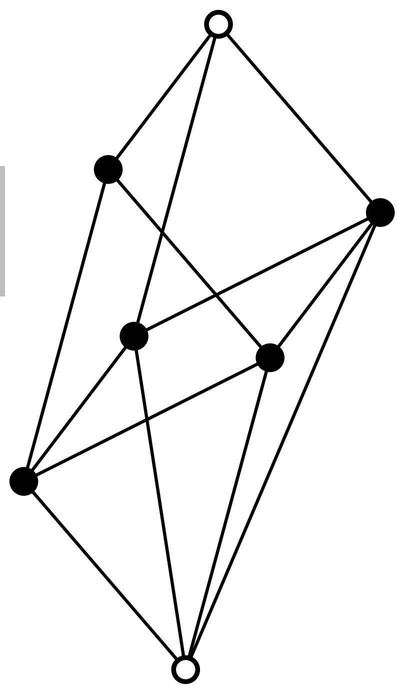

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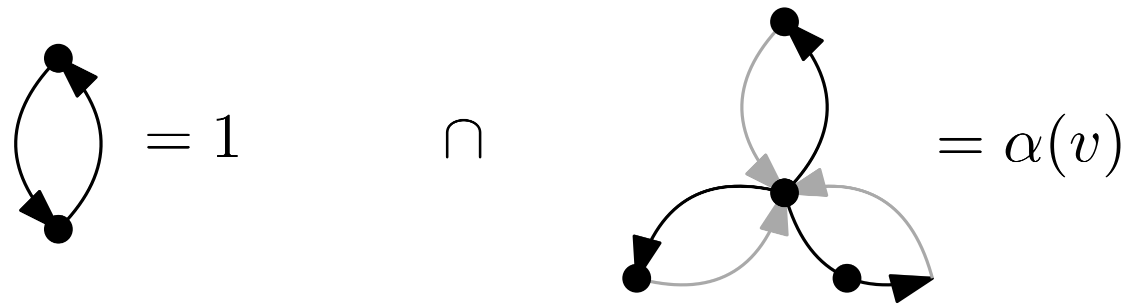

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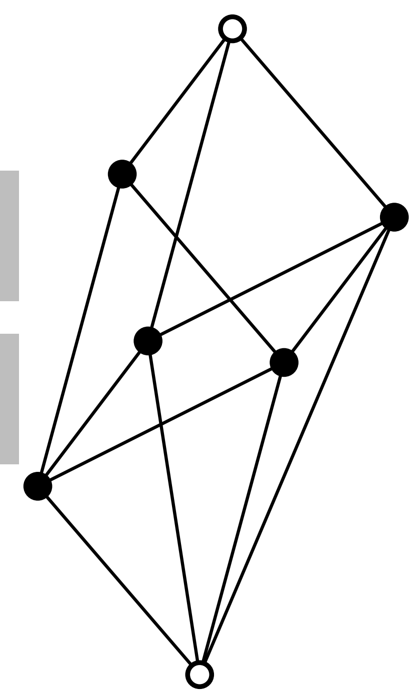

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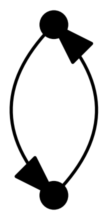
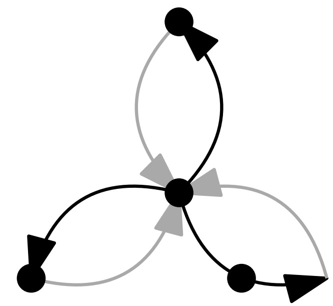
 $= 1$ $\cap$  $= \alpha(v)$

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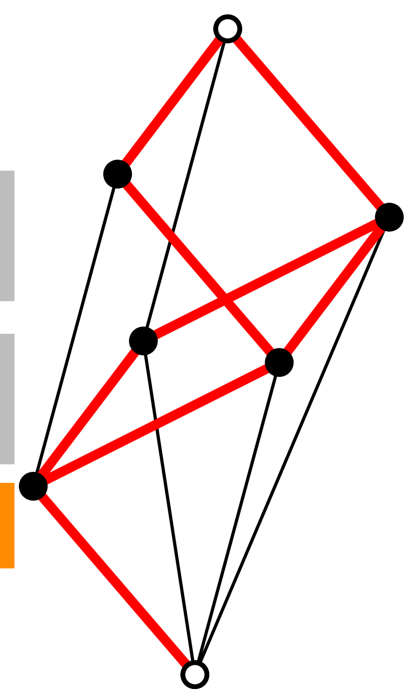

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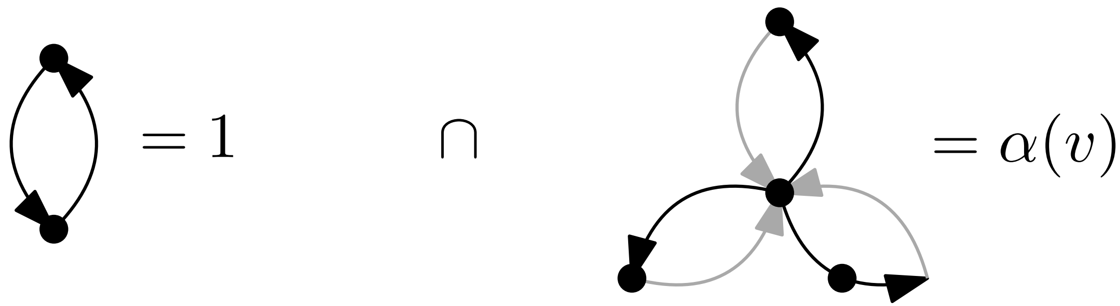

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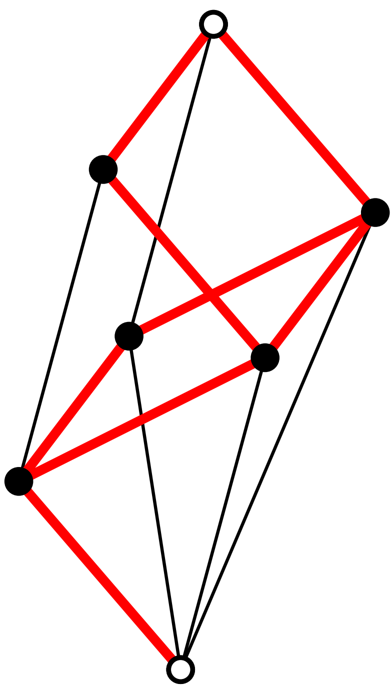

and now for something completely different

Generating orientations

given small input G generate large output L such that each $\ell \in L$ is produced exactly once

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acyclic orientations	$\mathcal{O}(n)$	$\mathcal{O}(n^3)$(Squire '98)
	$\mathcal{O}(n+m)$	$\mathcal{O}(nm)$(Barbosa, Szwarcfiter '99)
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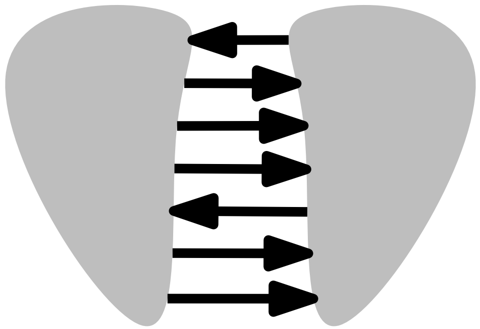

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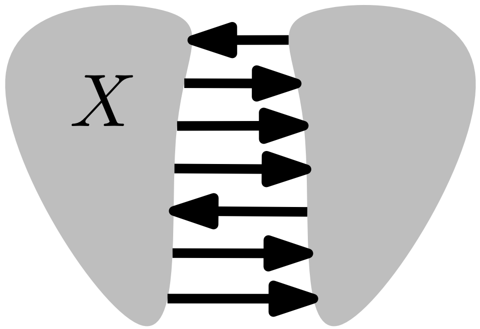

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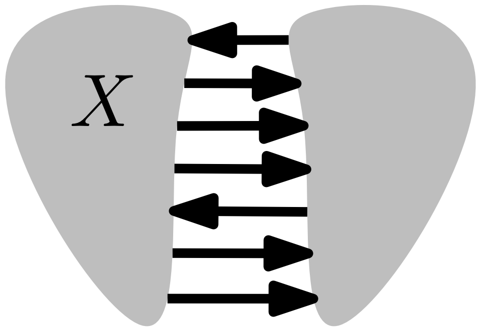

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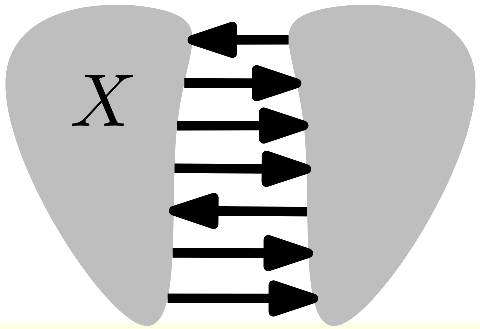

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G has k-connected orientations iff G $2k$-edge connected

Thm[Frank '86]: Given $G = (V, E \cup A)$ mixed, decide in polytime $p(|G|)$ if G has k-connected orientation (*submodular flows*)

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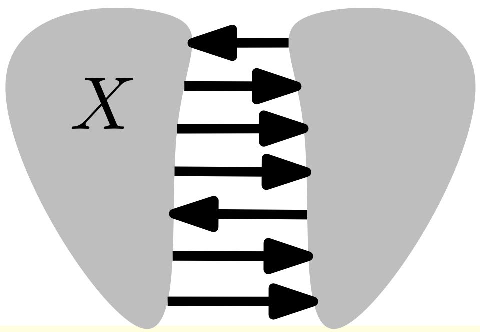

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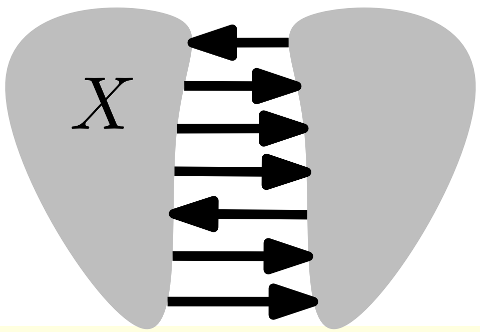

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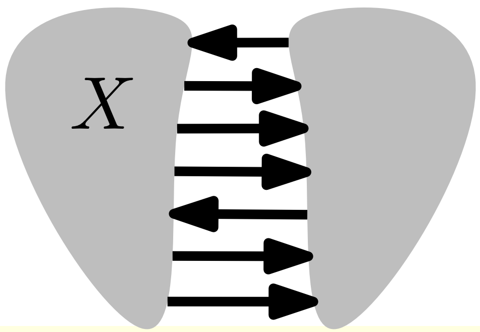

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m

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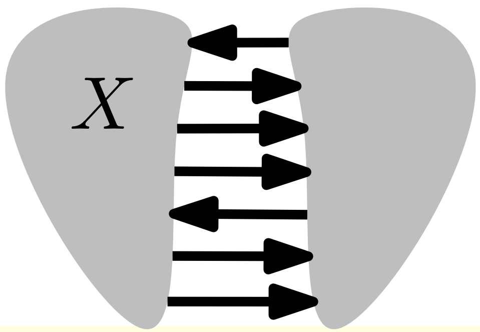

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$\delta^+(X) \geq k$ for all $\emptyset \neq X \subsetneq V$

$\geq k$ arc disjoint directed paths from x to y for all $x, y \in V$ $\rightsquigarrow \lambda(x,y) \geq k$

Thm[Nash-Williams '60];
G has k-connected orientations iff G $2k$-edge connected

Thm[Frank '86]: Given $G = (V, E \cup A)$ mixed, decide in polytime $p(|G|)$ if G has k-connected orientation (*submodular flows*)

$G = (V, E)$

m

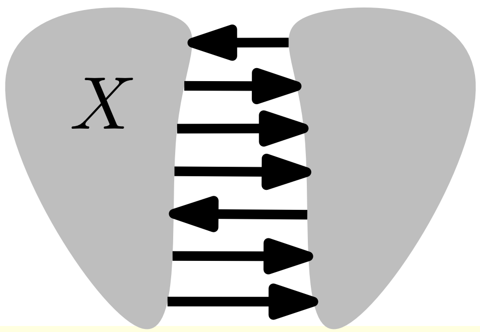
$G = (V, E \setminus a \cup a^+)$ $G = (V, E \setminus a \cup a^-)$

$\rightsquigarrow$ delay: $\mathcal{O}(mp(|G|))$

$p(|G|) \in \mathcal{O}(kn^2(\sqrt{kn} + k^2 \log(\frac{n}{k}))$
(Gabow '93) ...involved

k-connected orientations

must remove $\geq k$ arcs to destroy strong connectivity



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Us: simple algo with delay $\mathcal{O}(knm^2)$ amortized $\mathcal{O}(m^2)$

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Generating α-orientations

$$O_\alpha(G) = \{D \mid \delta^+(v) = \alpha(v) \forall_{v \in V}\}$$

things: $D, D' \in O_\alpha(G) \iff D \setminus D'$ disjoint union of directed cycles

D k-connected $\iff$ D' k-connected

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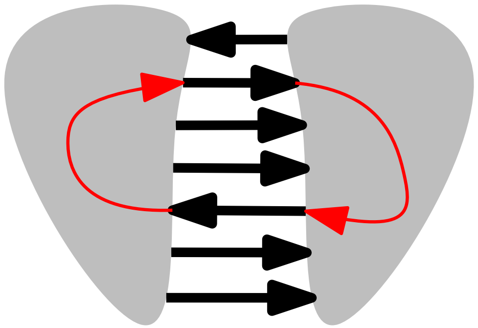

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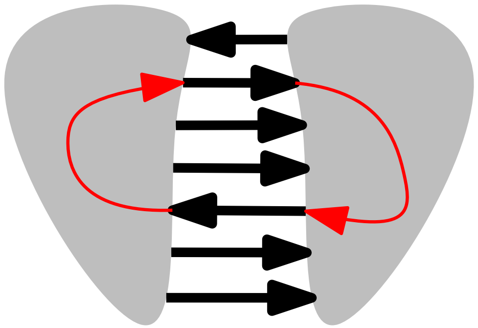

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take the next $a = (uv) \notin F$

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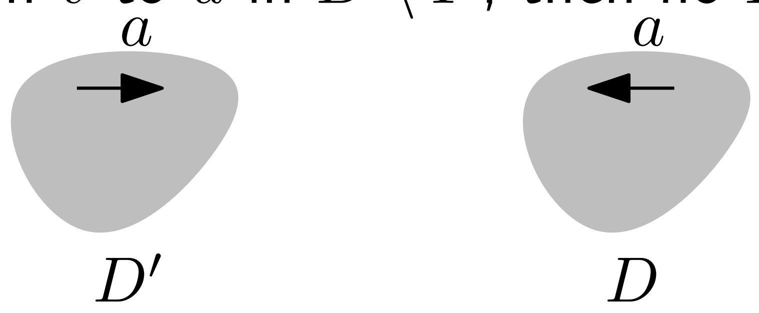

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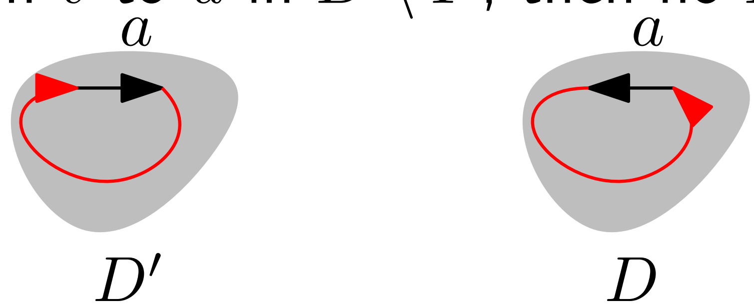

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$\rightsquigarrow$ obtain every element of O_α

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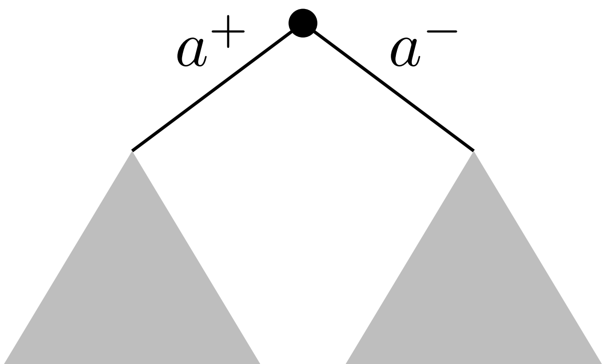

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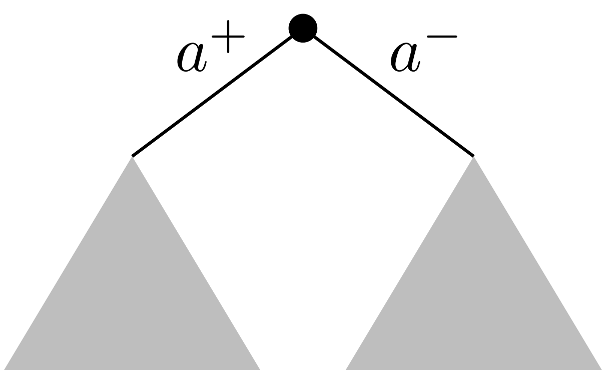

$\rightsquigarrow$ obtain every element of O_α

$\rightsquigarrow$ exactly once

$\rightsquigarrow$ delay $\mathcal{O}(m\text{BFS}) = \mathcal{O}(m^2)$

Generating k-connected αs

things: Reversing directed P_{xy}
$\rightsquigarrow \lambda(x, y) := \lambda(x, y) - 1$ and $\lambda(x', y') \geq \min(\lambda(x', y'), \lambda(x, y) - 1)$

Generating k-connected αs

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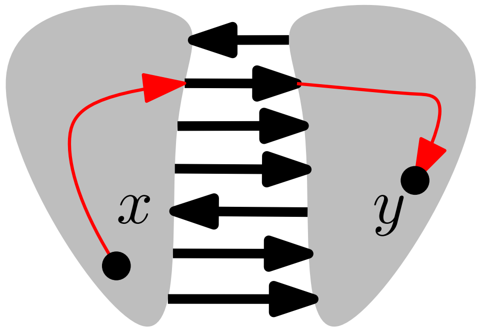

decrease $\delta^+(X)$ for $x \in X \not\ni y$

Menger: decrease $\lambda(x,y)$

$\lambda(x',y')$ decreased to at most $\lambda(x,y) - 1$

Generating k-connected αs

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L1: P_{xy} *flippable* : $\iff$ D^P k-connected $\iff$ $\lambda(x,y) > k$

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L2: Compute in $\mathcal{O}(k\text{BFS})$ if $\lambda(x,y) \geq k$

Find directed P_{xy}, reverse, iterate...

Generating k-connected αs

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$\implies \exists \delta^+_D(y) > \delta^+_{D'}(y)$ s.th (y,x) flippable

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consider the maximal such Xs

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consider the maximal such Xs they are pairwise disjoint 'cause otherwise

$k + k \geq \delta^+(X) + \delta^+(X') \geq \delta^+(X \cup X') + \delta^+(X \cap X') \geq k + k$

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$$\implies \delta^+(X \cup X') = k \implies \text{contradict maximality.}$$

Generating k-connected αs

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r=edges not contained in any X, s=number of Xs

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r=edges not contained in any X, s=number of Xs

$$r = ks + \sum_{z \notin \cup X} \delta^+_D(z) < \sum_X \delta^+_{D'}(X) + \sum_{z \notin \cup X} \delta^+_{D'}(z) = r$$

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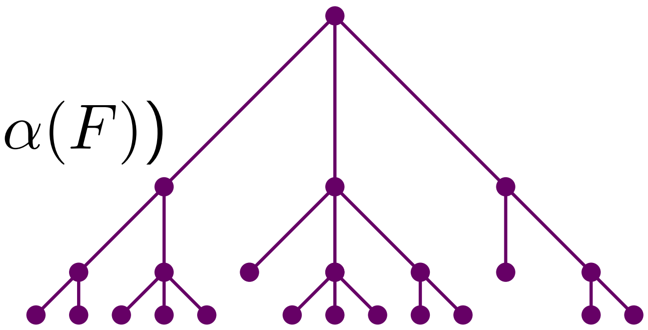

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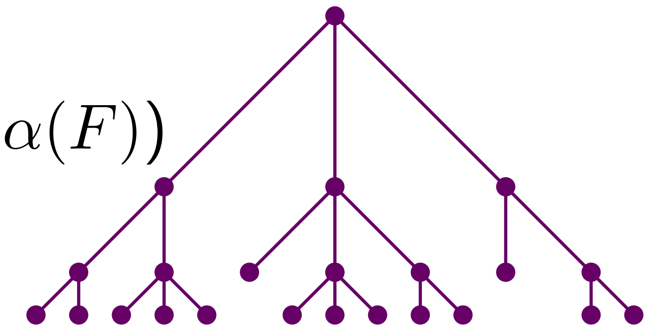

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L1+2 $\implies \mathcal{O}(nk\text{BFS})$

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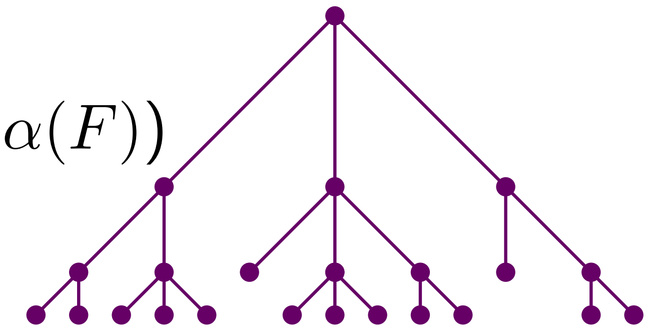
L2: Compute in $\mathcal{O}(k\text{BFS})$ if $\lambda(x,y) \geq k$

L3: D, D' k-connected, $\delta^+_D(x) < \delta^+_{D'}(x)$
$\implies \exists \delta^+_D(y) > \delta^+_{D'}(y)$ s.th (y,x) flippable

gen(D,V) : input: D k-connected, $F \subseteq V$
output: all k-connected αs fixing $\alpha(F)$
take the next $x \notin F$ (if $F = V$ output $\alpha(F)$)

- gen$^+(D,F,x)$
- gen$^-(D,F,x)$
- gen$(D,F \cup x)$



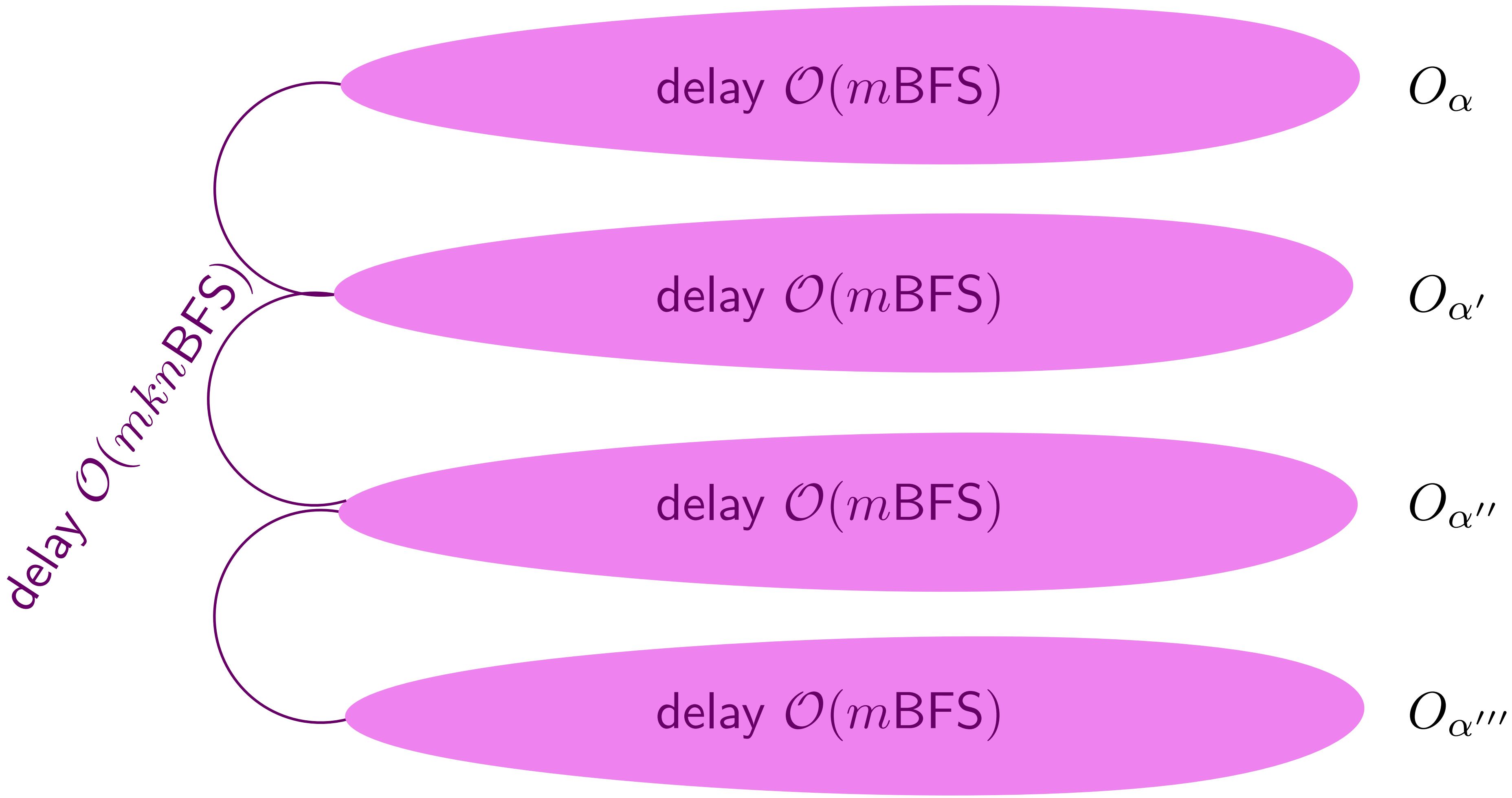
gen$^+(D,V,x)$: output: all k-connected αs with $\alpha(x) > \delta^+(x)$
if there is $y \notin F$ with (y,x) flippable, then

- gen$^+(D^P,F,x)$
- gen$(D^P, F \cup x)$

L1+2 $\implies \mathcal{O}(nk\text{BFS})$

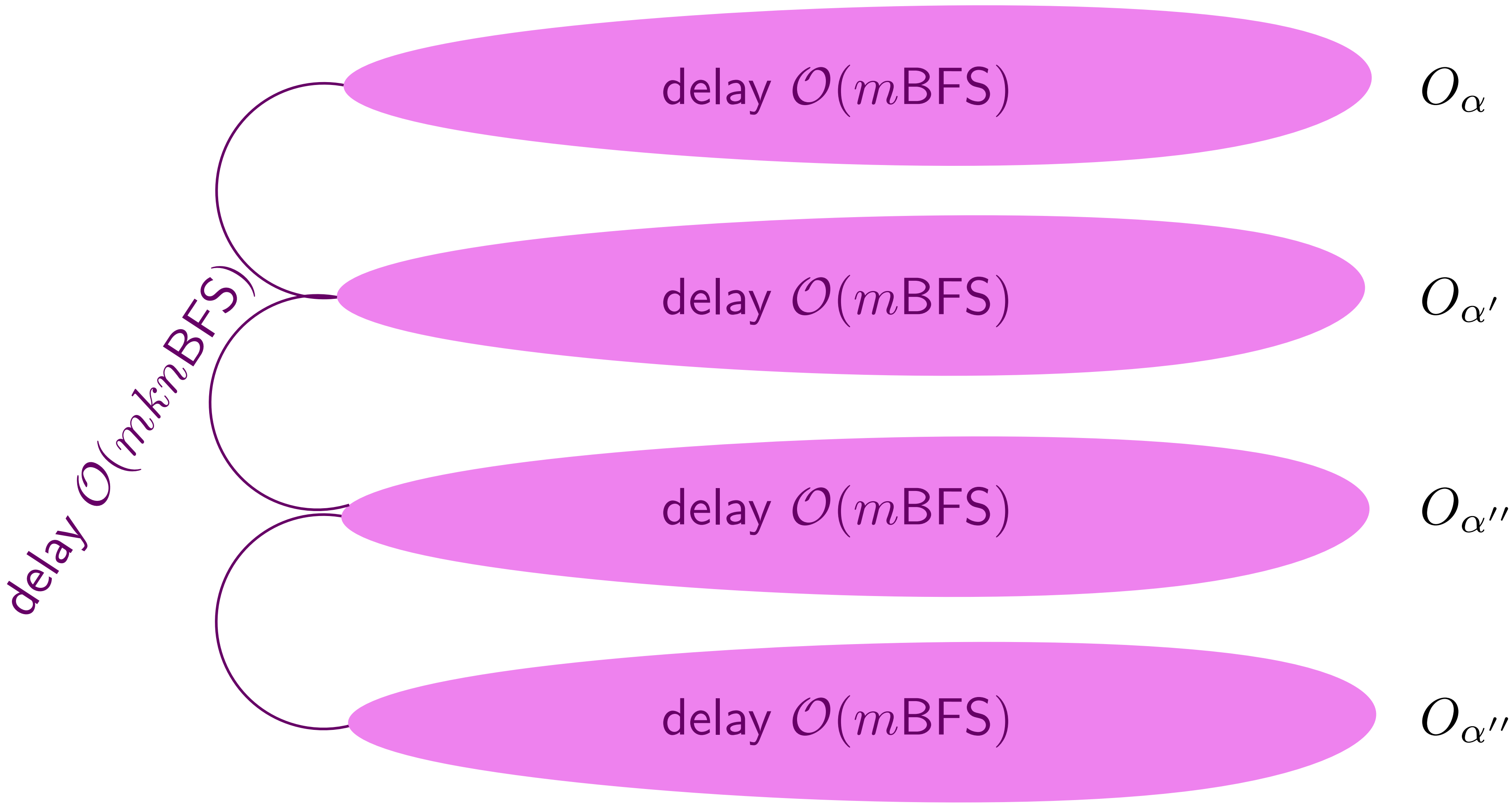
L3 $\implies$ generate all

Last slide



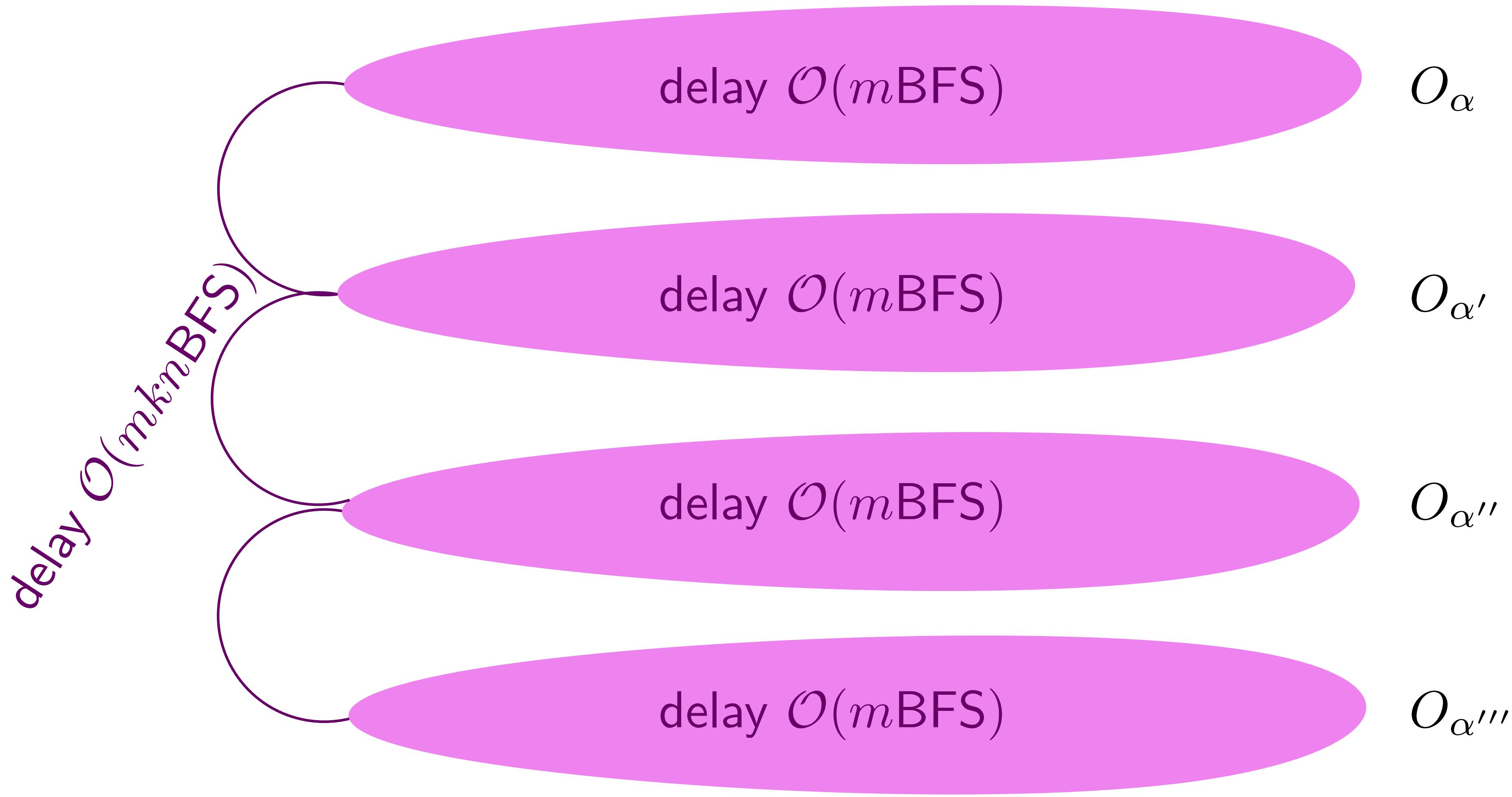
delay $\mathcal{O}(m\text{BFS})$
O_α
delay $\mathcal{O}(m\text{BFS})$
$O_{\alpha'}$
delay $\mathcal{O}(m\text{BFS})$
$O_{\alpha''}$
delay $\mathcal{O}(m\text{BFS})$
$O_{\alpha'''}$
delay $\mathcal{O}(mkn\text{BFS})$

Last slide



$|O_\alpha(G)| \geq (k-1)n + 2 \rightsquigarrow$ amortized time $O(m^2)$

Last slide



$|O_\alpha(G)| \geq (k-1)n + 2 \rightsquigarrow$ amortized time $O(m^2)$

small generating sets for $O_\alpha(G)$?

generating sets k-vertex-connected orientations?